

ADVANCED MATH CLASSES

@ Think Academy, Najafgarh

(For Half Yearly Exam. Paper)

Class : XII

Syllabus : Chapter 1 to 8

MATHEMATICS-041

For 2026-27 Exams

Maximum Marks : 80

Time : 3 Hours

General Instructions :

- All questions are compulsory.
- This question paper consists of **38 questions** divided into **five sections A, B, C, D and E**.
- Section A comprises of **20 questions of one mark** each (from Q01 - 20).
Section B comprises of **05 questions of two marks** each (from Q21 - 25).
Section C comprises of **06 questions of three marks** each (from Q26 - 31).
Section D comprises of **04 questions of five marks** each (from Q32 - 35).
Section E comprises of **03 Case-study** based question with **sub-parts** (from Q36 - 38).
- There is no overall choice. However, **internal choice** has been provided in
 - **02 Questions of Section B**
 - **03 Questions of Section C**
 - **02 Questions of Section D**
 - **02 Questions of Section E**

You have to attempt only one of the alternatives in all such questions.

SECTION A

(Question numbers 01 to 20 carry **1 mark** each.)

Followings are **multiple choice questions**. Select the correct options (from **Q01 to 18**).

- Q01.** For two matrices A and B, given that $A^{-1} = \frac{1}{4}B$, then inverse of (4A) is
 (a) 4B (b) B (c) $\frac{1}{4}B$ (d) $\frac{1}{16}B$
- Q02.** If a matrix has 36 elements, the number of possible orders it can have, is
 (a) 13 (b) 3 (c) 5 (d) 9
- Q03.** The number of discontinuities of the function f given by $f(x) = \begin{cases} x+2, & \text{if } x < 0 \\ e^x, & \text{if } 0 \leq x < 1 \\ 2-x, & \text{if } x \geq 1 \end{cases}$ is
 (a) 0 (b) 1 (c) 2 (d) 3
- Q04.** If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is
 (a) 7 (b) 6 (c) 8 (d) 18
- Q05.** If $\int_0^2 2e^{2x} dx = \int_0^a e^x dx$, the value of 'a' is
 (a) 1 (b) 2 (c) 4 (d) $\frac{1}{2}$
- Q06.** Let $\sin^{-1}(x) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$. Then the value of x is

- (a) $\frac{1}{8}$ (b) $\frac{1}{2}$ (c) 8 (d) $\frac{1}{4}$

Q07. $\int_a^b f(x)dx$ is equal to

- (a) $\int_a^b f(a-x)dx$ (b) $\int_a^b f(a+b-x)dx$ (c) $\int_a^b f(x-(a+b))dx$ (d) $\int_a^b f((a-x)+(b-x))dx$

Q08. A function $f: R_+ \rightarrow R$ (where R_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is

- (a) one-one but not onto (b) onto but not one-one
(c) both one-one and onto (d) neither one-one nor onto

Q09. If $A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \\ 0 & 3 & -2 \end{bmatrix}$, then the value of $|A \text{ adj}(A)|$ is

- (a) -1 (b) 1 (c) 2 (d) 3

Q10. Principal value of $\tan^{-1}\left(\tan \frac{5\pi}{6}\right)$ is

- (a) $\frac{5\pi}{6}$ (b) $-\frac{\pi}{6}$ (c) $\frac{\pi}{6}$ (d) $-\frac{5\pi}{6}$

Q11. Let $y = f\left(\frac{1}{x}\right)$ and $f'(x) = x^3$. What is the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}$?

- (a) $-\frac{1}{64}$ (b) $-\frac{1}{32}$ (c) -32 (d) -64

Q12. The derivative of $\sin(x^2)$ w.r.t. x , at $x = \sqrt{\pi}$ is

- (a) 1 (b) -1 (c) $-2\sqrt{\pi}$ (d) $2\sqrt{\pi}$

Q13. $\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix} =$

- (a) $2x^3$ (b) 2 (c) 0 (d) $2x^3 - 2$

Q14. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^3 \cos^2 x \, dx$ is equal to

- (a) 0 (b) -1 (c) 1 (d) 2

Q15. The greatest integer function defined by $f(x) = [x]$, $1 < x < 3$ is **not** differentiable at $x =$

- (a) 0 (b) 1 (c) 2 (d) $\frac{3}{2}$

Q16. For the function $f(x) = x^3$, $x = 0$ is a point of

- (a) local maxima (b) local minima
(c) non-differentiability (d) inflexion

Q17. $\int \frac{x-3}{(x-1)^3} e^x \, dx$ is equal to

- (a) $\frac{2e^x}{(x-1)^3} + C$ (b) $\frac{-2e^x}{(x-1)^2} + C$ (c) $\frac{e^x}{(x-1)} + C$ (d) $\frac{e^x}{(x-1)^2} + C$

- Q18.** If X , Y and XY are matrices of order 2×3 , $m \times n$ and 2×5 respectively, then the number of elements in matrix Y is
 (a) 6 (b) 10 (c) 15 (d) 35

Followings are **Assertion (A) - Reason (R) based questions** (from Q19 to 20).

Read the following statements carefully to mark the correct option out of the options given below.

- (a) A is true, R is true; R is a correct explanation for A.
 (b) A is true, R is true; R is not a correct explanation for A.
 (c) A is true, R is false.
 (d) A is false, R is true.

- Q19. Assertion (A) :** For the matrix given by $A = \begin{bmatrix} 1 & \cos \theta & 1 \\ -\cos \theta & 1 & \cos \theta \\ -1 & -\cos \theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$, we shall have $|A| \in [2, 4]$.

Reason (R) : $\cos \theta \in [-1, 1]$, $\forall \theta \in [0, 2\pi]$.

- Q20. Assertion (A) :** $\cos^{-1}\left(\cos \frac{13\pi}{6}\right)$ is equal to $\frac{\pi}{6}$.

Reason (R) : The range of the principal value branch of the function $y = \cos^{-1} x$ is $[0, \pi]$.

SECTION B

(Question numbers 21 to 25 carry 2 marks each.)

- Q21.** (a) Find the value of $\cos^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \operatorname{cosec}^{-1}(-2)$. Use principal values.

OR

- (b) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.

- Q22.** Find a matrix A such that $A \begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}$.

- Q23.** (a) Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

OR

- (b) Show that the function given as $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

- Q24.** For a given square matrix A of order 3 such that $A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$, show that $A^3 = A^{-1}$.

- Q25.** A relation R on set $A = \{1, 2, 3, 4, 5\}$ is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$.

Check whether the relation R is transitive.

SECTION C

(Question numbers 26 to 31 carry 3 marks each.)

- Q26.** If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then find the value of k .

- Q27.** (a) Check whether the relation S in the set of all real numbers ($\mathbb{R}$) defined by $S = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive.

OR

(b) A function f is defined from $\mathbb{R} \rightarrow \mathbb{R}$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function $f(x)$. Hence, check whether function $f(x)$ is one-one and onto or not.

Q28. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

(b) If $y = (\tan x)^x$, then find $\frac{dy}{dx}$.

Q29. A relation R is defined on $\mathbb{N} \times \mathbb{N}$ (where $\mathbb{N}$ is the set of natural numbers) as $(a, b)R(c, d) \Leftrightarrow a - c = b - d$. Show that R is an equivalence relation.

Q30. (a) Find $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$.

OR

(b) Evaluate $\int \frac{dx}{\cos x \sqrt{\cos 2x}}$.

Q31. Find the intervals in which the function $f(x) = \frac{4 \sin x}{2 + \cos x} - x$, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

SECTION D

(Question numbers 32 to 35 carry 5 marks each.)

Q32. (a) Evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$.

OR

(b) Evaluate $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x) dx$.

Q33. Solve the following linear programming problem graphically.

Maximise : $z = 4x + 3y$,

Subject to the constraints : $x + y \leq 800$, $2x + y \leq 1000$, $x \leq 400$; $x, y \geq 0$.

Q34. (a) Solve the following system of equations, using matrices

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2, \quad \text{where } x, y, z \neq 0.$$

OR

(b) If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that $A'A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$.

Q35. Using integration, determine the area of the region bounded by the curve $y = \sqrt{4-x^2}$, the lines $x = -\sqrt{2}$ and $x = \sqrt{3}$ and the x -axis.

SECTION E

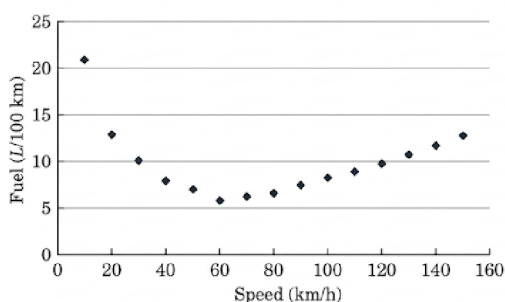
This section contains **three Case-study based questions** (from Q36 to 38).

This section contains **three Case-study / Passage based questions**.

First two questions have **three sub-parts** (i), (ii) and (iii) of marks 1, 1 and 2 respectively.

Third question has **two sub-parts** of **2 marks** each.

- Q36.** Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



Relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is

given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.

On the basis of above information, answer the following questions.

- (i) Find F , when $V = 40$ km/h.

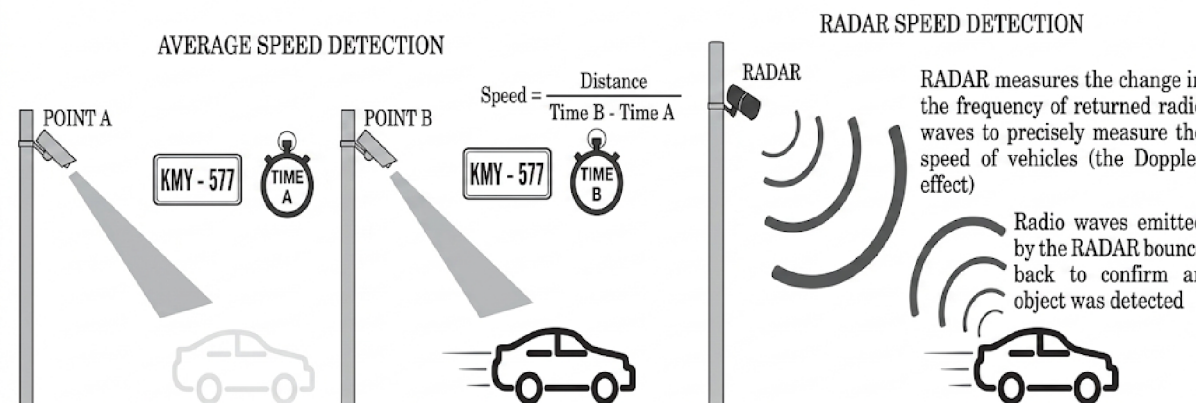
- (ii) Find $\frac{dF}{dV}$.

- (iii) Find the speed V for which fuel consumption F is minimum.

OR

- (iii) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.

- Q37.** The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.



A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s.

At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ .

On the basis of above information, answer the following questions.

- (i) Express θ in terms of height of the camera installed on the pole and x .

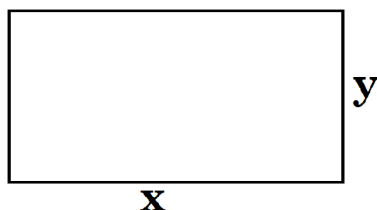
- (ii) Find $\frac{d\theta}{dx}$.

- (iii) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole.

OR

(iii) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car.

- Q38.** An architect is developing a plot of land for a commercial complex. When asked about the dimensions of the plot, he said that if the length is decreased by 25 m and the breadth is increased by 25 m, then its area increases by 625 m^2 . If the length is decreased by 20 m and the breadth is increased by 10 m, then its area decreases by 200 m^2 . Refer the figure.

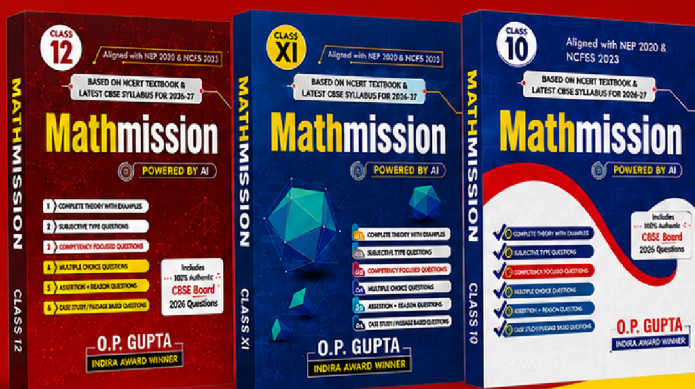


On the basis of the above information, answer the following questions.

- Formulate the linear equations in x and y to represent the given information.
- Find the dimensions of the plot of land by matrix method.

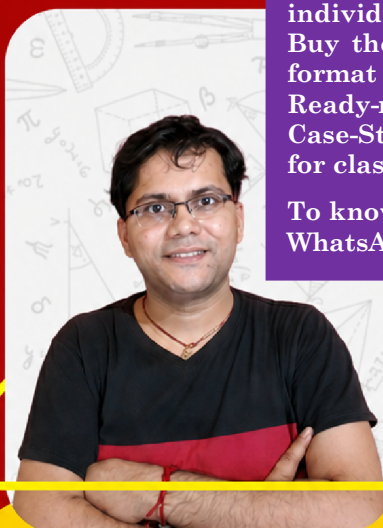
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With Regards

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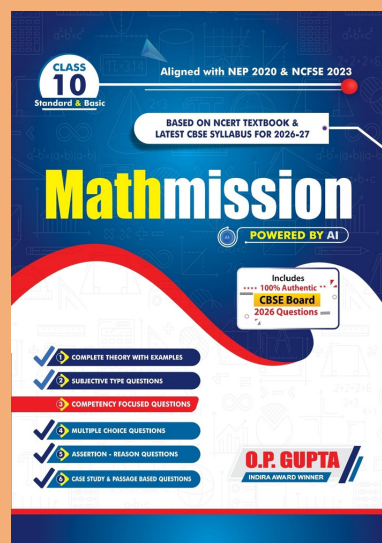
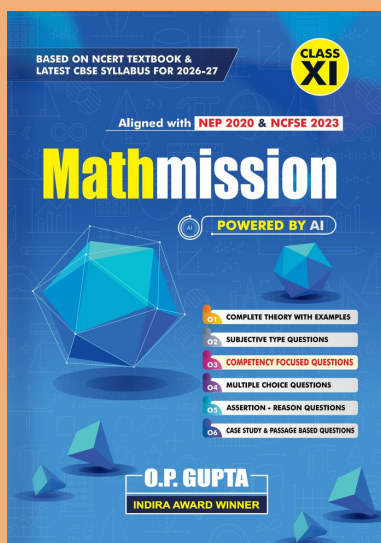
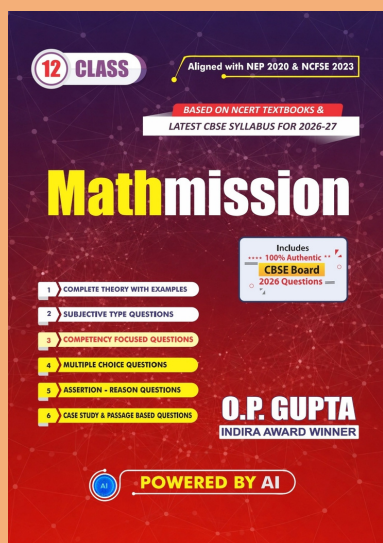
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